LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2013**

MT 3501/MT 3500 – ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date : 06/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART – A

Answer ALL questions: **(10 × 2 = 20)**

1. Evaluate $\int_0^1 \int_0^1 \int_0^1 dx\, dy\, dz.$
2. Define Beta and Gamma function.
3. Form a partial differential equation by eliminating a and b from $z= ax +by +a^2+ b^2$.
4. Solve pq=1.
5. For, $\varphi= x^2 +y – z - 1$ find $\nabla \varphi$ at (1,0 , 0).
6. Show that the vector $\vec{F}= 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is solenoidal.
7. Find L $(3e^{5t} + 5 \cos t)$
8. Find $L^{-1} \left(\frac{2s}{s^2+25} + \frac{1}{s^3} \right)$
9. Find $\varphi(729)$.
10. State Fermat's theorem.

PART – B

Answer any FIVE questions: **(5 × 8 = 40)**

11. Evaluate by changing the order of the integration $\int_{-a}^{a} \int_0^{\sqrt{a^2-y^2}} x\, dx\, dy.$
12. Express $\int_0^1 x^m (1-x^n)^p dx$ in terms Gamma functions
13. Solve $x^2p^2 + y^2q^2 = z^2$.
14. Solve $y^2p + x^2q = x^2y^2z^2$.

15. Find (i) $L(t\sin^2 t)$ (ii) $L(\cos^4 t)$.

16. Find $L^{-1}\left(\frac{s-1}{2s^2+s+6}\right)$.

17. Prove that if $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ show that $\nabla \times (r^n\vec{r}) = 0$.

18. Show that 8 th power of any number is of the form 17m or 17m± 1.

PART – C

Answer any THREE questions: **(2 × 20 =40)**

19. (a) Evaluate $\iiint xyz\, dz\, dy\, dx$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Show that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

20. (a) Obtain the complete and singular solution of $\frac{z}{pq} = \frac{x}{q} + \frac{y}{p} + \sqrt{pq}$.

(b) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

21 (a) Verify Gauss divergence theorem for $\bar{F} = x\hat{i} + y\hat{j} + z\hat{k}$ taken over the region bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0$ and $z = a$.

(b) Show that 18!+1 is divisible by 437. (12+8)

22. Using Laplace transform solve $\frac{d^2y}{dt^2} + 4\frac{dy}{dx} + 13y = 2e^{-t}$ given that $y = 0; \dfrac{dy}{dt} = 1$ when $t = 0$.

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